Data Mining
Association Analysis: Basic Concepts and Algorithms

Lecture Notes for Chapter 6

정보보호 및 지능형 IoT 연구실
Information Security & Intelligent IoT

Contents

Rule Generation

Rule Generation from frequent itemset

Given a frequent itemset L, find all non-empty subsets $f \subset L$ such that $f \rightarrow L - f$ satisfies the minimum confidence requirement

- If {A,B,C,D} is a frequent itemset, candidate rules:

ABC →D,	ABD →C,	ACD →B,	BCD →A,
A →BCD,	B →ACD,	C →ABD,	D →ABC
AB →CD,	AC → BD,	AD → BC,	BC →AD,
BD →AC,	CD →AB,		

If |L| = k, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \varnothing$ and $\varnothing \rightarrow L$)

Rule Generation from frequent itemset

How to efficiently generate rules from frequent itemsets?

- **신뢰도**(confidence)**는** anti-monotone **성질을 가지지 않는다**. → Apriori **특성 사용이 어려움**

  c(ABC →D) can be larger or smaller than c(AB →D)

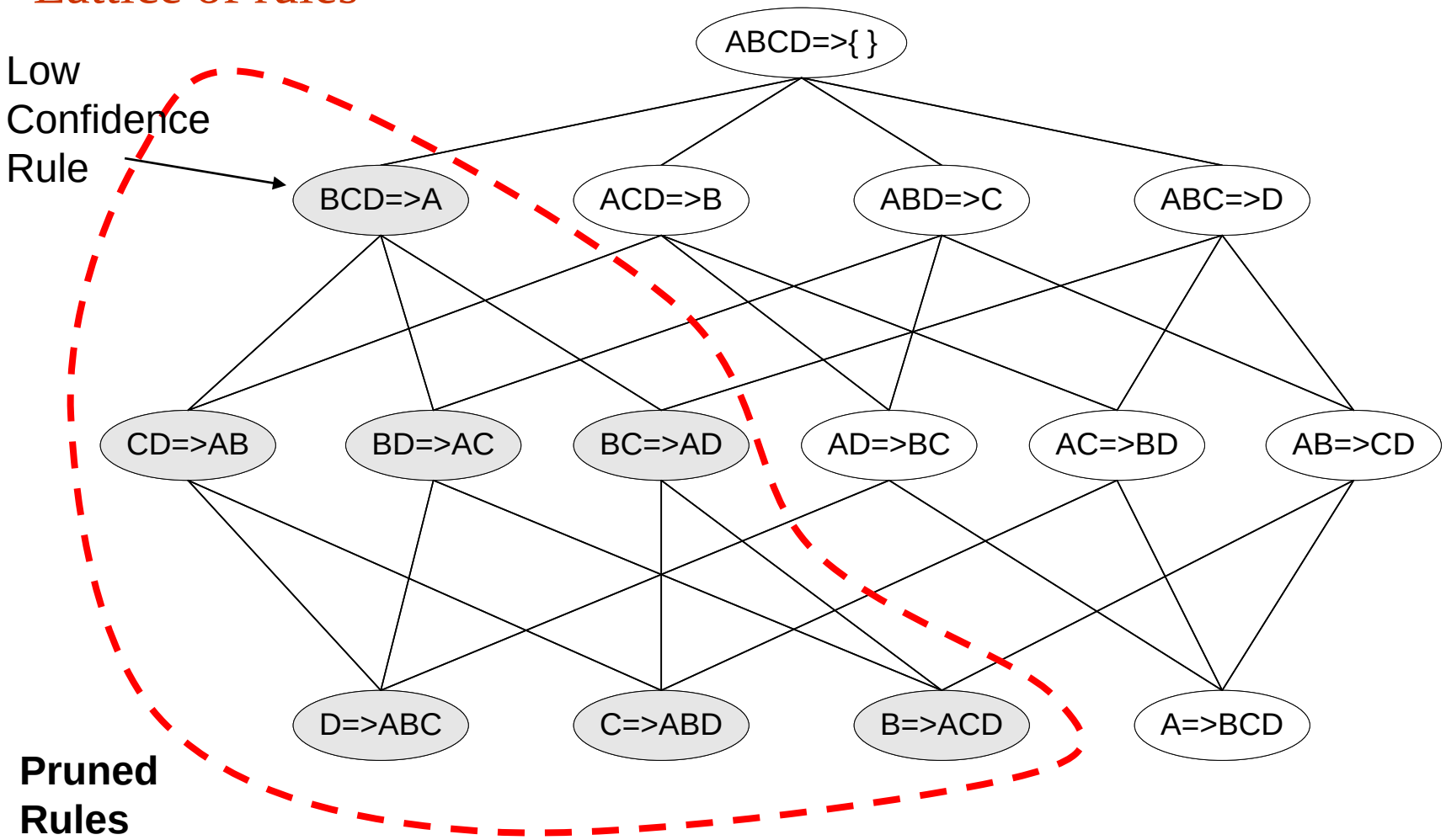
- **그러나, 동일한 항목집합에서 생성된 규칙에 대해서는** anti-monotone **성질이 성립**
- (That is, confidence is anti-monotone w.r.t number of items on the RHS of the rule, or monotone w.r.t. the LHS of the rule)
- e.g., L = {A,B,C,D}:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

$$c(ABC \rightarrow D) = \frac{\sigma(\{A,B,C,D\})}{\sigma(\{A,B,C\})}$$
$$c(AB \rightarrow CD) = \frac{\sigma(\{A,B,C,D\})}{\sigma(\{A,B\})}$$
$$c(A \rightarrow BCD) = \frac{\sigma(\{A,B,C,D\})}{\sigma(\{A\})}$$

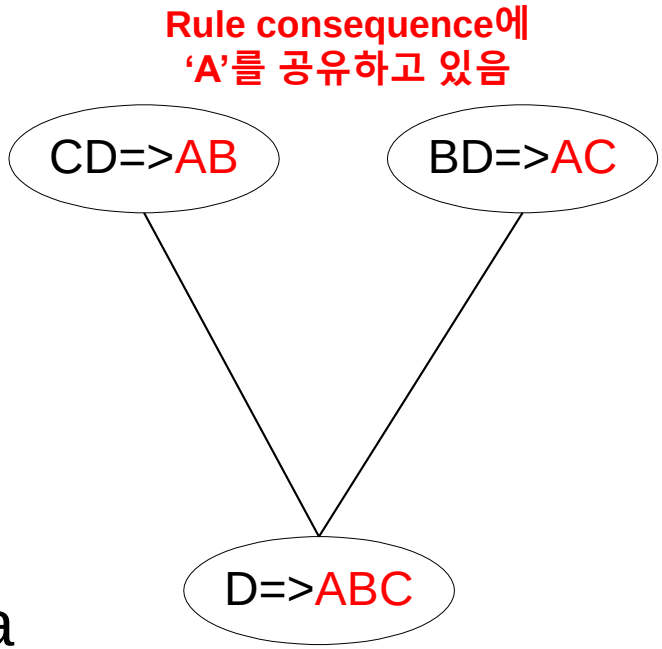
Rule Generation for Apriori Algorithm

Lattice of rules



Rule Generation for Apriori Algorithm

Candidate rule is generated by merging two rules that share the same prefix in the rule consequent



join(CD→AB,BD→AC) would produce the candidate rule D → ABC

Prune rule D →ABC if there exists a subset (AD→BC) that does not have high confidence

Essentially, we are doing Apriori on the RHS

참고 **c(ABC → D) ≥ c(AB → CD) ≥ c(A → BCD)**

Contents

Maximal itemset/closed itemset

Maximal Frequent Itemset

An itemset is maximal frequent(최대 빈발 항목집합) if none of its immediate supersets are frequent → 결국 빈발한 가장 긴 항목 찾기. 포함집합(immediate superset) 어느것도 빈발하지 않아야 함

That is, this is a frequent itemset which is not contained in another frequent itemset.

찾는 방법

- 먼저 Infrequent와 frequent itemset 사이의 border에 있는 frequent itemset 찾음
- 모든 immediate supersets을 찾음
- 만약 immediate superset 모두가 frequent 하지 않으면, 해당 itemset은 maximal frequent함
  - ✓ 예: **Items: a, b, c, d, e**
  - ✓ **Frequent Itemset: {a, b, c}**
  - ✓ **{a, b, c, d}, {a, b, c, e}, {a, b, c, d, e} are not Frequent Itemset.**
  - ✓ **Maximal Frequent Itemsets: {a, b, c}**

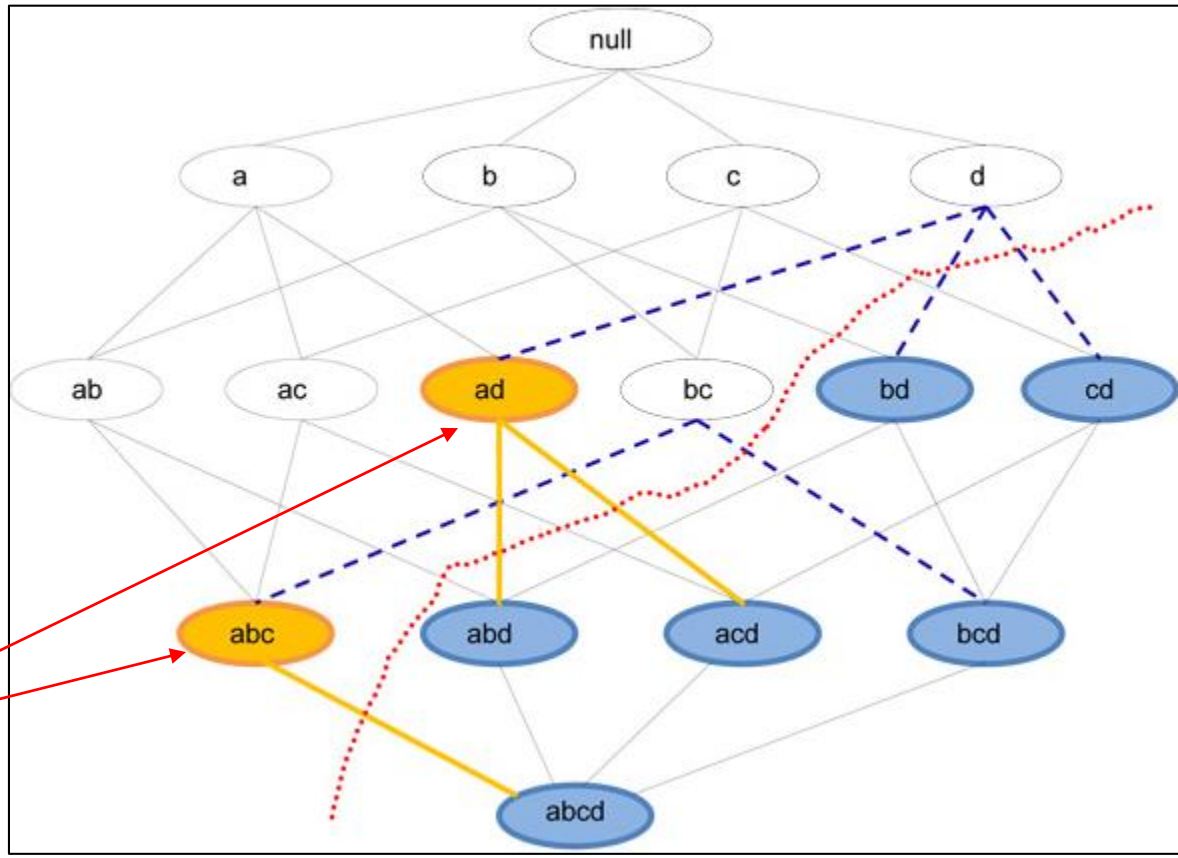
Maximal frequent itemset은 아주 긴 빈발 항목집합을 만들 때 유용함

- 일반적으로 짧은 항목집합은 규칙으로서 큰 의미가 없는 경우가 많음
- 반면에, 긴 항목집합은 대개가 surprise한 연관규칙을 생성할 수 있음

Maximal Frequent Itemset

Maximal frequent itemset 찾는 예 1:

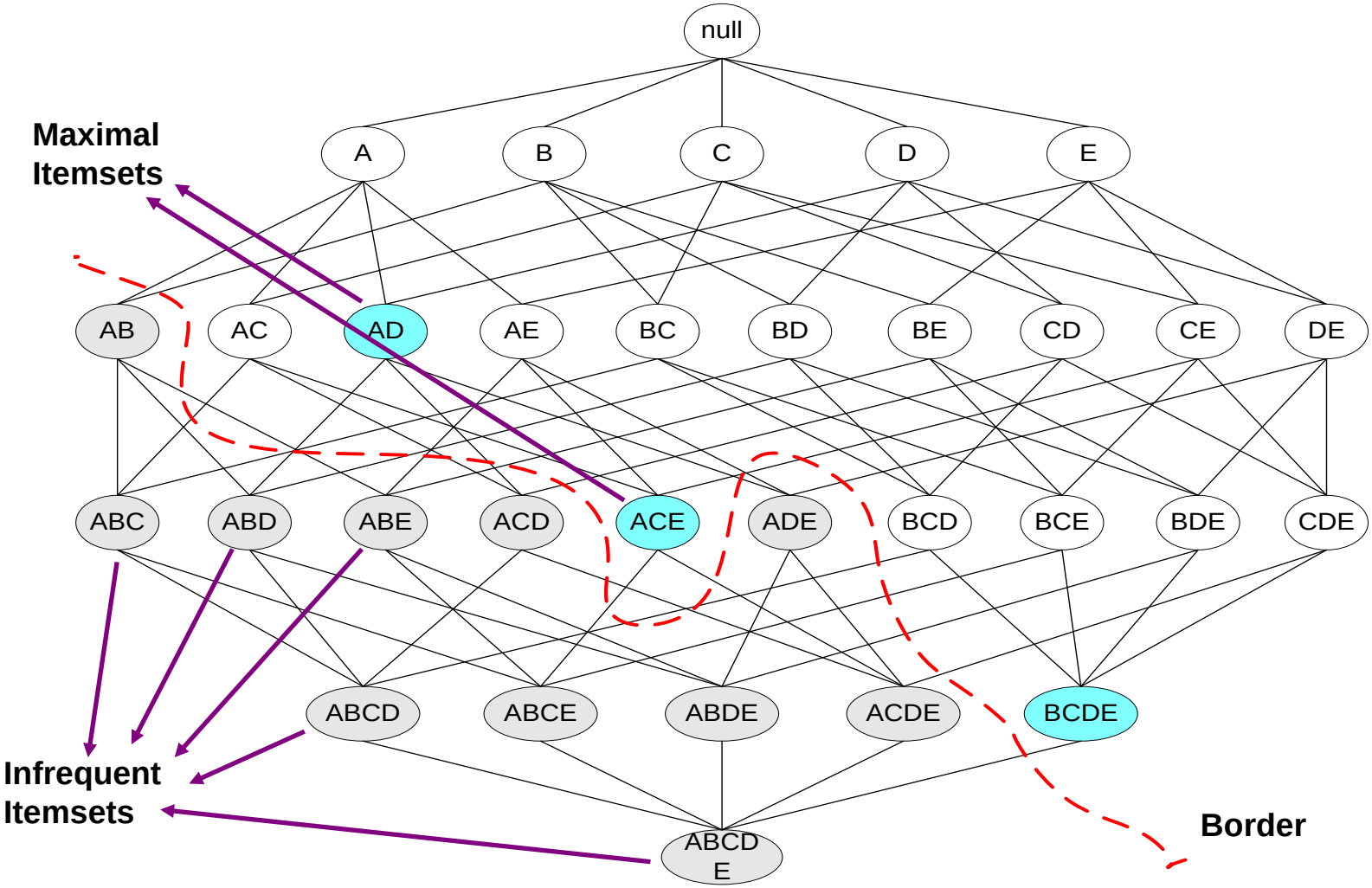
- 먼저 Infrequent와 frequent itemset 사이의 border에 d, bc, ad, abc frequent itemset 이 있음을 확인함
- 이들 itemset의 immediate superset을 찾음
  - d의 superset으로 ad, bd, cd 가 있는데, ad는 frequent임. → **d는 maximal 이 되지 못함**
  - bc는 abc와 bcd를 superset 으로 갖는데, abc가 frequent함 → bc는 maximal되지 못함
  - ad와 abc의 superset은 모두 infrequent 함 → ad와 abc는 모두 maximal임



빈발한 가장 긴 항목

Maximal Frequent Itemset

Maximal frequent itemset 찾는 예 2:



Closed Itemset

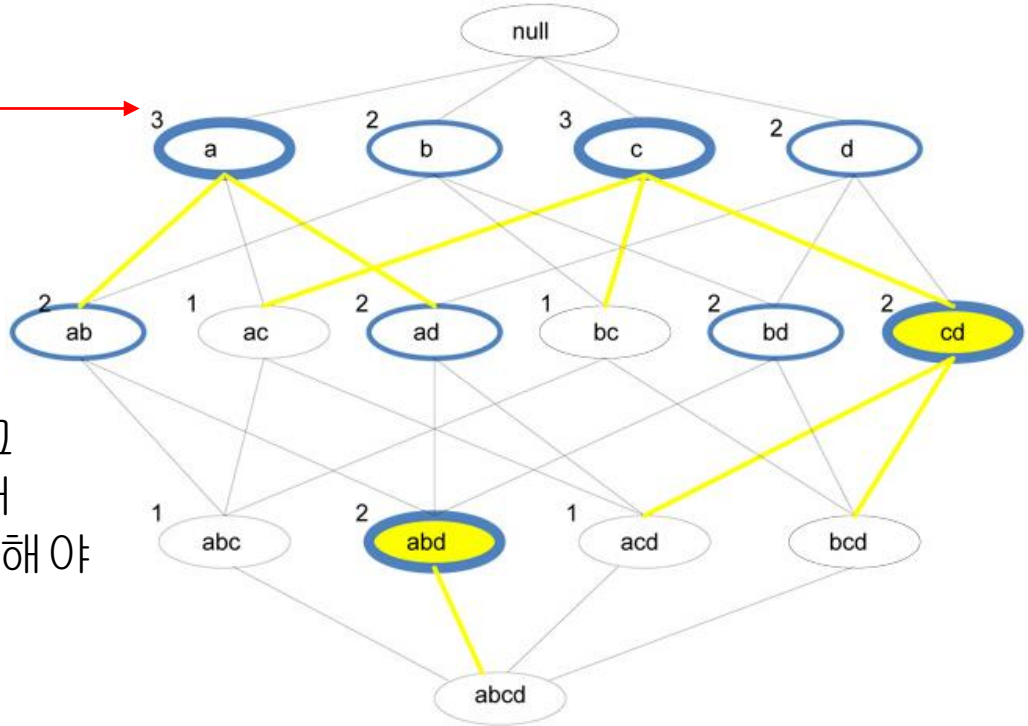
An itemset is closed if none of its immediate supersets has the same support as the original itemset → 그 항목의 수퍼패턴이 동일한 support(지지도) 값을 가지면 안됨 → 그러면 그 수퍼패턴 항목이 closed인지 확인필요

That is, this is a set of items which is as large as it can possibly be without losing any transactions

Closed itemset이 frequent 하면 closed frequent itemset임

예 closed frequent itemset 찾는

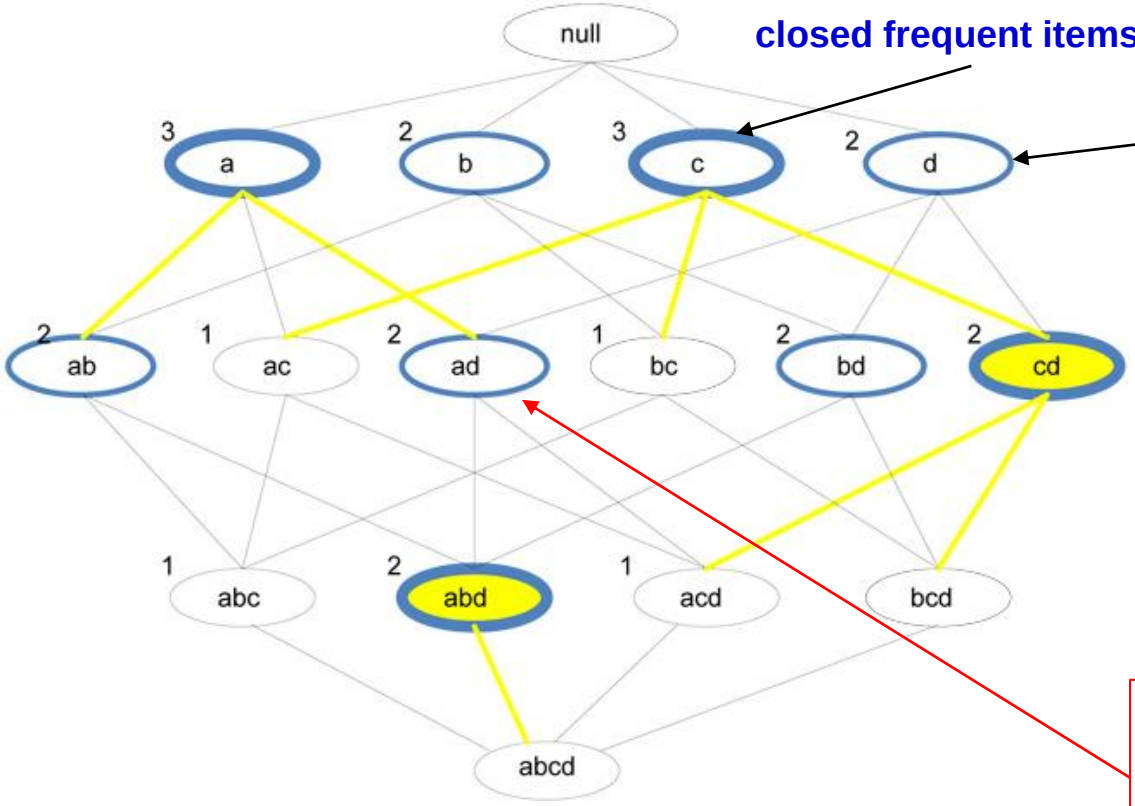
지지도 값: transaction 3곳에 이 item 이 있음을 의미



- 1. frequent해야 하며
- 2. 해당 항목의 superset(수퍼패턴)과 동일한 지지도를 가지면 안됨. 즉, superset과 동일한 지지도를 가지면, 그 superset으로 옮겨가서, 다시 한단계 더 superset을 파악하여 closed 여부 확인해야

Closed Itemset

예 closed frequent itemset 찾는 방법



closed frequent itemset

frequent itemset

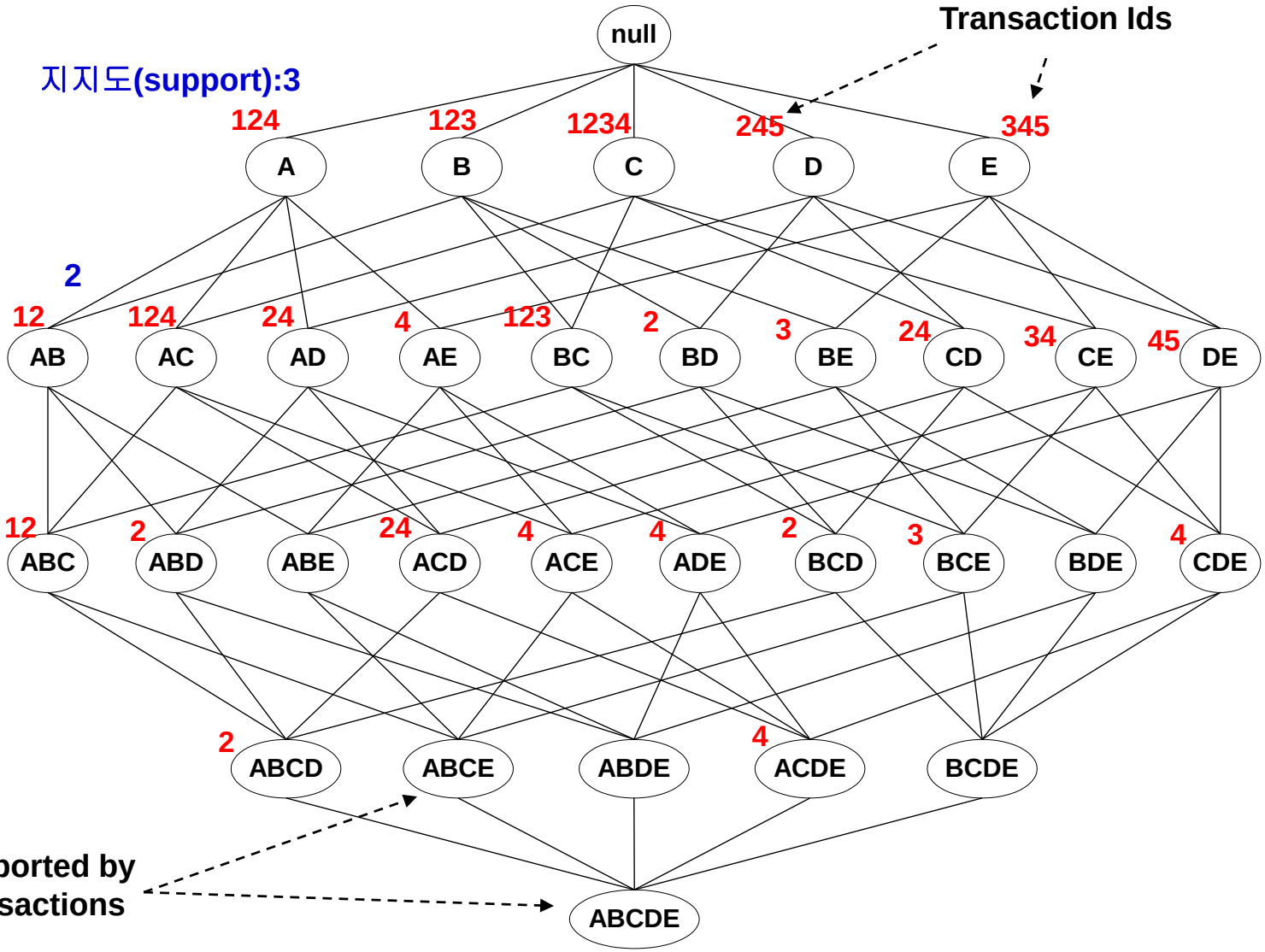
- c는 closed frequent itemset임. C의 superset인 ac, bc, cd는 3보다 작은 support 값을 가지므로
- 왼쪽 예제에서 총 9개의 frequent itemset 이 존재하며, 이중에서 4개가 closed frequent itemset 임

- ad는 frequent itemset 이지만 superset인 abd와 동일한 support 값을 가지므로 closed 아님

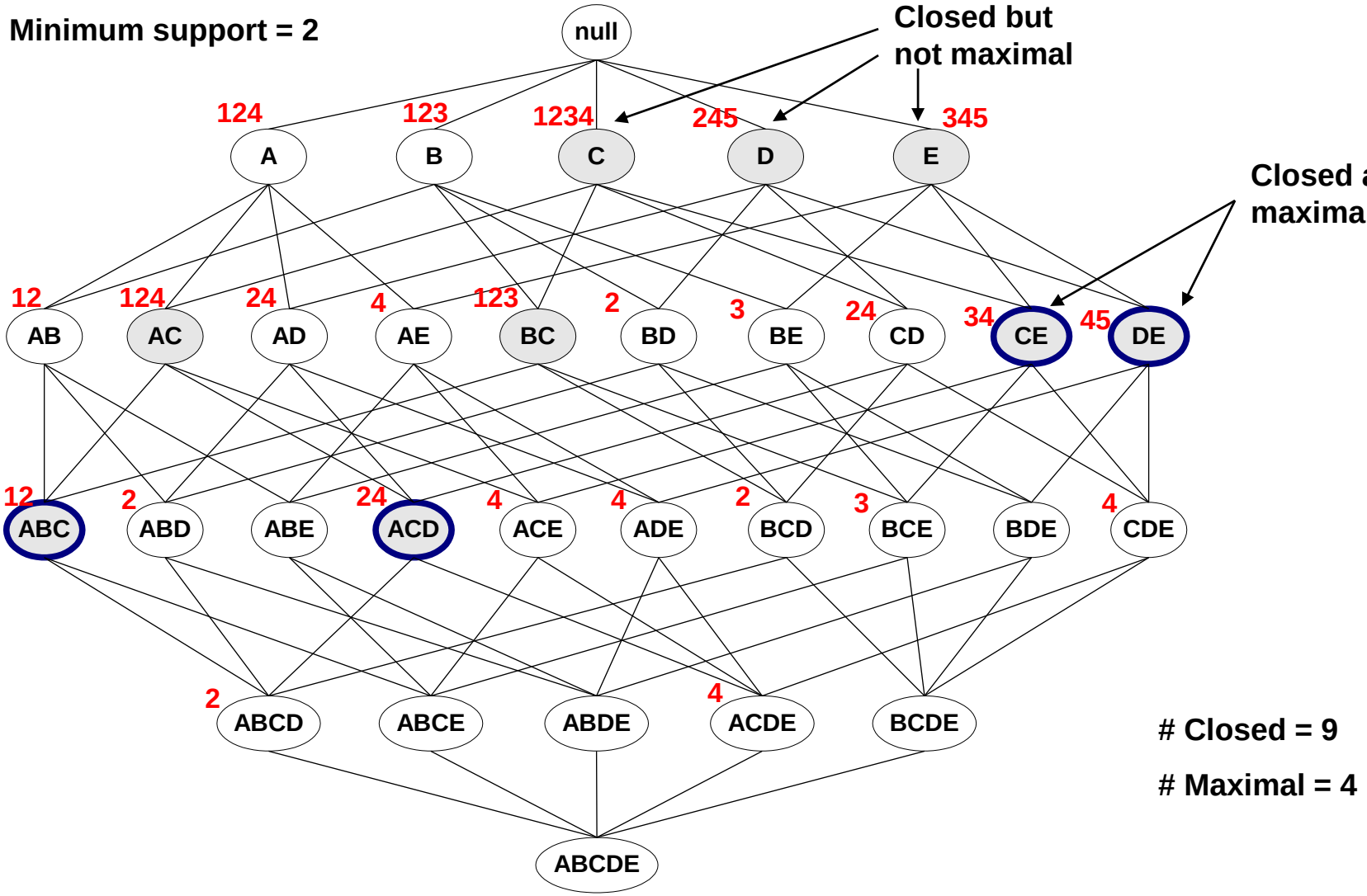
- 파란색 circle은 frequent itemset 임
- 파란색 두꺼운 circle 은 closed frequent itemset 임(closed는 superset과 동일한 support값을 가지지 않아야 함)
- 노란색 색칠된 circle은 maximal frequent itemset 임

Maximal vs Closed Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



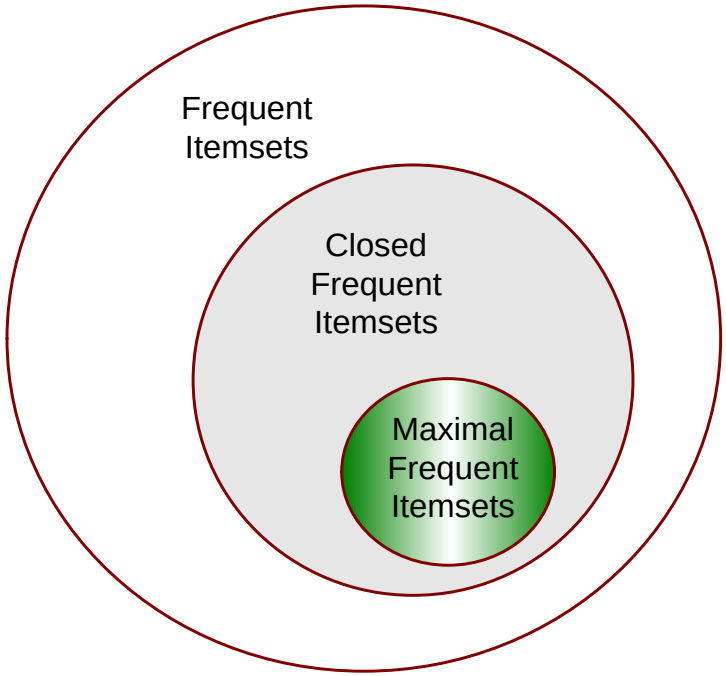
Maximal vs Closed Frequent Itemsets



Example of Closed Itemsets

Refer to http://simpledatamining.blogspot.com/2015/02/closed-itemsets.html

Maximal vs Closed Itemsets



- it is important to point out the relationship between frequent itemsets, closed frequent itemsets and maximal frequent itemsets.
- Closed and maximal frequent itemsets are subsets of frequent itemsets but maximal frequent itemsets are a more compact representation because it is a subset of closed frequent itemsets.
- The diagram to the right shows the relationship between these three types of itemsets.

Contents

연관 패턴의 평가

연관 규칙 평가(Pattern Evaluation)

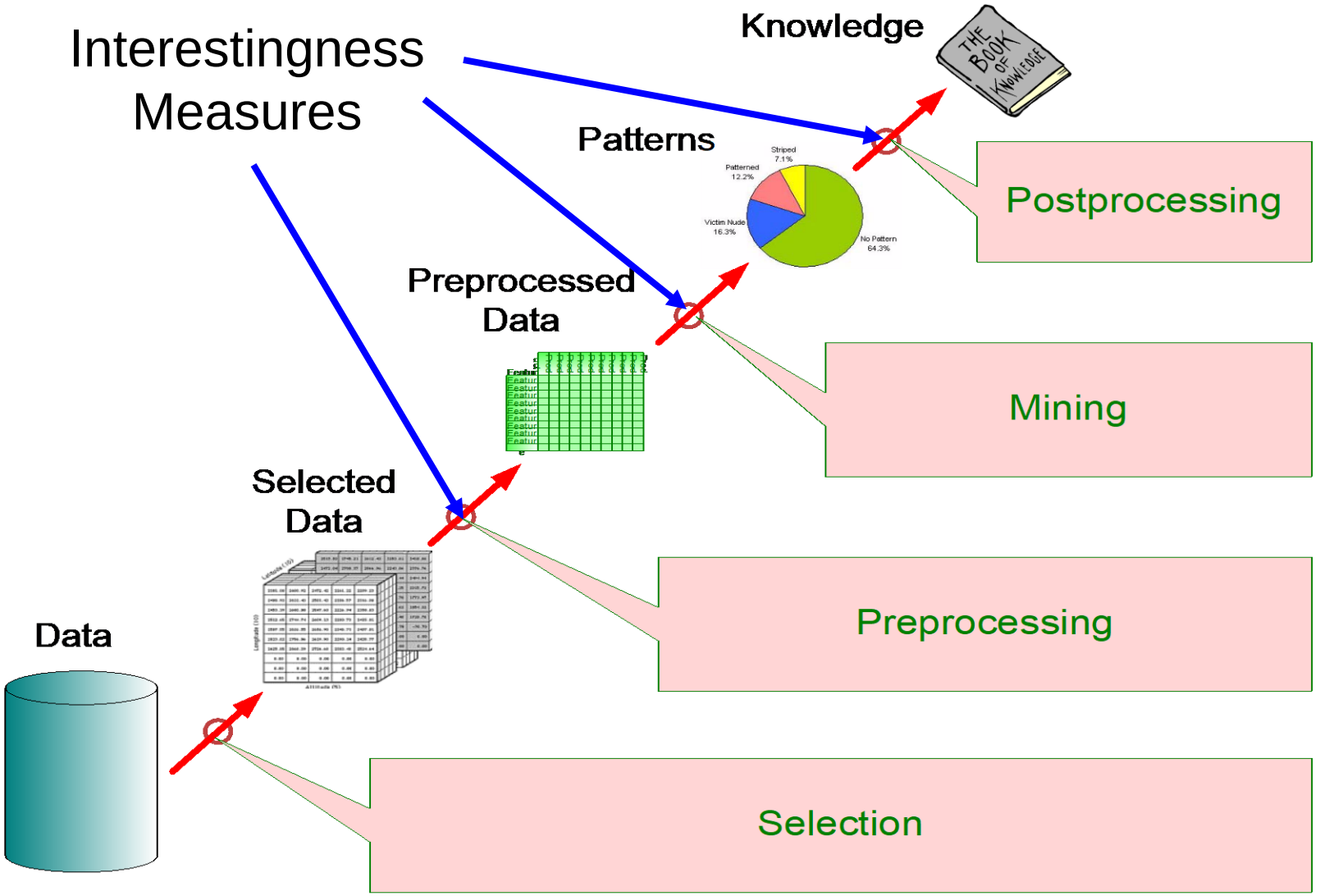
연관 규칙 생성 알고리즘은 너무 많은 연관 규칙을 생성하는 경향이 있음

- 생성된 많은 규칙은 유용하지 않음(uninteresting or redundant)
- 예를 들어, {A, B, C} → {D}와 {A, B} → {D}가 동일한 지지도/신뢰도를 갖는다면, 이들 두 규칙은 redundant 함

Interestingness measures(유용성 척도)는 유도된 규칙을 제거하거나 순위를 매기는데(prune or rank) 사용됨

지지도와 신뢰도(support & confidence)도 유용성 척도에 속함

유용성 척도 활용 시점


Interestingness Measures
Knowledge
THE BOOK OF KNOWLEDGE
Patterns
Striped 7.1%
Patterned 12.2%
Victim Nude 16.3%
No Pattern 64.3%
Postprocessing
Preprocessed Data
Mining
Selected Data
Preprocessing
Data
Selection

Computing Interestingness Measure

주어진 규칙 X → Y에 대해, 다음 분할표(contingency table)를 사용하여 다양한 유용성 척도를 계산할 수 있다

Contingency table for X → Y

	Y	$\overline{Y}$	
X	f_{11}	f_{10}	f_{1+}
$\overline{X}$	f_{01}	f_{00}	f_{o+}
	f_{+1}	f_{+0}	|T|

f_{ij}는 support 즉, 빈도수 count 값을 의미함

f_{1+}는 결국 X에 대한 지지도 count를 의미함

f_{11}: support of X and Y
f_{10}: support of X and $\overline{Y}$
f_{01}: support of $\overline{X}$ and Y
f_{00}: support of $\overline{X}$ and $\overline{Y}$

X항목이 transaction에 없는 경우 → $\bar{X}$

Used to define various measures

support, confidence, lift, Gini, J-measure, etc.

신뢰도의 단점(Drawback of Confidence)

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	65	15	80
	80	20	100

$$s(X \rightarrow Y) = \frac{\sigma(X \cup Y)}{N}$$

$$c(X \rightarrow Y) = \frac{\sigma(X \cup Y)}{\sigma(X)}$$

Association Rule: Tea → Coffee

- Support(Tea → Coffee) = 15/100 = 15%
- Confidence(Tea → Coffee) = s(Tea U Coffee)/s(Tea) = 15/20= 75%

- 위 신뢰도를 보고 Tea를 마시는 사람은 coffee를 마시는 경향이 있다고 볼지도 모름
- 하지만, 위 데이터를 보면 Tea를 마시든 마시지 않든 간에, coffee 를 마시는 사람의 비율은 원래 80%였음
- 즉, 규칙 Tea → Coffee를 통해, 어떤 사람이 차를 마신다는 정보를 통해 커피를 마시는 사람의 정보를 아는 것은 (75%라는 큰 신뢰도 값을 가짐에도) 큰 의미가 없음.

Statistical Independence

Population of 1000 students

- 600 students know how to swim (S)
- 700 students know how to bike (B)
- 420 students know how to swim and bike (S,B)

- $P(S \wedge B) = 420/1000 = 0.42$
- $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$

- $P(S \wedge B) = P(S) \times P(B)$ => Statistical independence
- $P(S \wedge B) > P(S) \times P(B)$ => Positively correlated
- $P(S \wedge B) < P(S) \times P(B)$ => Negatively correlated

Statistical-based Measures

Measures that take into account statistical dependence

$$Lift = \frac{P(Y \mid X)}{P(Y)}$$

$$Interest = \frac{P(X,Y)}{P(X)P(Y)}$$

$$Note: \frac{P(Y \mid X)}{P(Y)} = \frac{\frac{P(X,Y)}{P(X)}}{P(Y)} = \frac{P(X,Y)}{P(X)P(Y)}$$

Lift와 Interest는 equivalent함

연관 규칙 평가(Pattern Evaluation)

Lift of an association rule: X → Y, lift = P(Y/X)/P(Y)

- If Lift > 1, then X and Y appear more often together than expected
  - this means that the occurrence of X has a positive effect on the occurrence of Y or that X is positively correlated with Y.
- If Lift < 1 then, X and Y appear less often together than expected
  - this means that the occurrence of X has a negative effect on the occurrence of Y or that X is negatively correlated with Y
- If Lift = 1, then X and Y are independent.
  - this means that the occurrence of X has almost no effect on the occurrence of Y

Example: Lift/Interest

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

$\Rightarrow$ Lift = P(Coffee/Tea)/P(Coffee) = 0.75/0.9= **0.8333 (< 1, therefore, the Lift is suggesting a slight negative correlation b/w tea drinkers and coffee drinkers)**

There are lots of measures proposed in the literature

Some measures are good for certain applications, but not for others

What criteria should we use to determine whether a measure is good or bad?

What about Apriori-style support based pruning? How does it affect these measures?

#	Measure	Formula
1	ϕ-coefficient	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha-1}{\alpha+1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa (κ)	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i,B_j)\log\frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i)\log P(A_i), -\sum_j P(B_j)\log P(B_j))}$
8	J-Measure (J)	$\max\left(P(A,B)\log(\frac{P(B\|A)}{P(B)}) + P(A\overline{B})\log(\frac{P(\overline{B}\|A)}{P(\overline{B})}),\right.$ $\left. P(A,B)\log(\frac{P(A\|B)}{P(A)}) + P(\overline{A}B)\log(\frac{P(\overline{A}\|B)}{P(\overline{A})})\right)$
9	Gini index (G)	$\max\Big(P(A)[P(B\|A)^2+P(\overline{B}\|A)^2]+P(\overline{A})[P(B\|\overline{A})^2+P(\overline{B}\|\overline{A})^2]$ $-P(B)^2-P(\overline{B})^2,$ $P(B)[P(A\|B)^2+P(\overline{A}\|B)^2]+P(\overline{B})[P(A\|\overline{B})^2+P(\overline{A}\|\overline{B})^2]$ $-P(A)^2-P(\overline{A})^2\Big)$
10	Support (s)	$P(A,B)$
11	Confidence (c)	$\max(P(B\|A), P(A\|B))$
12	Laplace (L)	$\max\left(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2}\right)$
13	Conviction (V)	$\max\left(\frac{P(A)P(\overline{B})}{P(A\overline{B})}, \frac{P(B)P(\overline{A})}{P(B\overline{A})}\right)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A,B)-P(A)P(B)$
17	Certainty factor (F)	$\max\left(\frac{P(B\|A)-P(B)}{1-P(B)}, \frac{P(A\|B)-P(A)}{1-P(A)}\right)$
18	Added Value (AV)	$\max(P(B\|A)-P(B), P(A\|B)-P(A))$
19	Collective strength (S)	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B\|A)-P(B), P(A\|B)-P(A))$