Homework #2

Discrete Mathematics- 2nd Semester 2018

Due: **Wednesday Oct, 24, 2018, 03:00 PM**

※ Solve the following problems.

1. Problem 1 in Exercises 17.1 (p. 806)

1. Let $f(x), g(x) \in \mathbf{Z}_7[x]$ where $f(x) = 2x^4 + 2x^3 + 3x^2 + x + 4$ and $g(x) = 3x^3 + 5x^2 + 6x + 1$. Determine $f(x) + g(x)$, $f(x) - g(x)$, and $f(x)g(x)$.

2. Problem 2 in Exercises 17.1 (p. 806)

2. Determine all of the polynomials of degree 2 in $\mathbf{Z}_2[x]$.

3. Problem 6 in Exercises 17.1 (p. 806)

6. For each of the following pairs $f(x)$, $g(x)$, find $q(x)$, $r(x)$ so that $g(x) = q(x)f(x) + r(x)$, where $r(x) = 0$ or degree $r(x) <$ degree $f(x)$.

a) $f(x), g(x) \in \mathbf{Q}[x]$, $f(x) = x^4 - 5x^3 + 7x$, $g(x) = x^5 - 2x^2 + 5x - 3$

b) $f(x), g(x) \in \mathbf{Z}_2[x]$, $f(x) = x^2 + 1$, $g(x) = x^4 + x^3 + x^2 + x + 1$

c) $f(x), g(x) \in \mathbf{Z}_5[x]$, $f(x) = x^2 + 3x + 1$, $g(x) = x^4 + 2x^3 + x + 4$

4. Problem 7 in Exercises 17.1 (p. 806)

7. a) If $f(x) = x^4 - 16$, find its roots and factorization in $\mathbf{Q}[x]$.

b) Answer part (a) for $f(x) \in \mathbf{R}[x]$.

c) Answer part (a) for $f(x) \in \mathbf{C}[x]$.

d) Answer parts (a), (b), and (c) for $f(x) = x^4 - 25$.

5. Problem 1 in Exercises 17.2 (p. 813)

1. Determine whether or not each of the following polynomials is irreducible over the given fields. If it is reducible, provide a factorization into irreducible factors.

a) $x^2 + 3x - 1$ over $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{C}$

b) $x^4 - 2$ over $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{C}$

c) $x^2 + x + 1$ over $\mathbf{Z}_3$, $\mathbf{Z}_5$, $\mathbf{Z}_7$

d) $x^4 + x^3 + 1$ over $\mathbf{Z}_2$

e) $x^3 + 3x^2 - x + 1$ over $\mathbf{Z}_5$

6. Problem 12 in Exercises 17.2 (p. 814)

12. For Example 17.9, determine which equivalence class contains each of the following:

a) $x^4 + x^3 + x + 1$

b) $x^3 + x^2 + 1$

c) $x^4 + x^3 + x^2 + 1$

참고) Example 17.9

Let $s(x) = x^2 + x + 1 \in \mathbf{Z}_2[x]$. Then

a) $[0] = [x^2 + x + 1] = \{0, x^2 + x + 1, x^3 + x^2 + x, (x + 1)(x^2 + x + 1), \ldots\}$
$= \{t(x)(x^2 + x + 1) | t(x) \in \mathbf{Z}_2[x]\}$

b) $[1] = \{1, x^2 + x, x(x^2 + x + 1) + 1, (x + 1)(x^2 + x + 1) + 1, \ldots\}$
$= \{t(x)(x^2 + x + 1) + 1 | t(x) \in \mathbf{Z}_2[x]\}$

c) $[x] = \{x, x^2 + 1, x(x^2 + x + 1) + x, (x + 1)(x^2 + x + 1) + x, \ldots\}$
$= \{t(x)(x^2 + x + 1) + x | t(x) \in \mathbf{Z}_2[x]\}$

d) $[x + 1] = \{x + 1, x^2, x(x^2 + x + 1) + (x + 1), (x + 1)(x^2 + x + 1)$
$+ (x + 1), \ldots\} = \{t(x)(x^2 + x + 1) + (x + 1) | t(x) \in \mathbf{Z}_2[x]\}$

7. Problem 16 (a) and (b) in Exercises 17.2(p. 814)

16. Let $s(x) = x^4 + x^3 + 1 \in \mathbf{Z}_2[x]$.

a) Prove that $s(x)$ is irreducible.

b) What is the order of the field $\mathbf{Z}_2[x]/(s(x))$?

8. GF(2^3) 유한체 상에서 (with x^3+x+1 minimal polynomial), 두개의 polynomial x^2+x와 x+1의 합과 곱을 계산하라.

- Thanks