Data Mining
Association Analysis: Basic Concepts and Algorithms

Lecture Notes for Chapter 6

정보보호 및 지능형 IoT 연구실
Information Security & Intelligent IoT

Contents

Association Rule Mining

Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Association Rules

{Diaper} → {Beer},
{Milk, Bread} → {Eggs,Coke},
{Beer, Bread} → {Milk},

Implication means co-occurrence, not causality!

Definition: Frequent Itemset

- **항목집합(Itemset)**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **지지도 카운트(Support count) (σ)**
  - 특정 항목 집합을 포함하는 transaction 수
  - 예: σ({Eggs}) = 1, σ({Milk, Bread,Diaper}) = 2
- **지지도(Support) s**
  - 항목집합이 나타나는 트랜잭션의 비율
  - 예: s{Eggs} = 1/5 = 0.2
    s({Milk, Bread, Diaper}) = 2/5
- **빈발항목집합(Frequent Itemset)**
  - 지지도가 주어진 임계치 minsup보다 큰 항목집합

2-itemset

4-itemset

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

만약 minsup = 0.3이라면, {Eggs}은 빈발하지 않으며, {Milk, Bread, Diaper}은 빈발함

Definition: Association Rule

- 연관규칙이란?
  - X와 Y가 항목집합 일때, $X \rightarrow Y$ 형태로 나타나는 함축 표현(implication expression)
  - 예: {Milk, Diaper} → {Bread}
- 연관규칙의 평가척도
  - 지지도(Support): s

$$s(X \rightarrow Y) = \frac{\sigma(X \cup Y)}{N}$$

    - X와 Y를 함께 포함하는 트랜잭션 비율
    - 규칙이 얼마나 중요하며, 유용한지를 알 수 있음 (낮은 지지도 갖는 규칙은 우연으로 볼 수 있음)
  - 신뢰도(Confidence): c

$$c(X \rightarrow Y) = \frac{\sigma(X \cup Y)}{\sigma(X)}$$

    - X를 포함한 트랜잭션 중에 Y가 나타나는 비율
    - 규칙이 얼마나 믿을 만 한가?
      (가정과 결론이 얼마나 타이트한 관련성 있는지를 나타냄)

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Support Count: $\sigma(X) = \left|\{t_i \mid X \subseteq t_i, t_i \in T\}\right|$

$T = \{t_1, t_2, t_3, \ldots, t_N\}$ 는 모든 transaction 집합

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - Support ≥ *minsup* threshold
  - Confidence ≥ *minconf* threshold

- 최소지지도(Minsup: minimum support)
- 최소신뢰도(Minconf: minimum confidence)
- 연관규칙 마이닝 = 왼쪽 조건을 만족하는 모든 규칙을 찾는 작업

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

  ⇒ Computationally prohibitive!

Mining Association Rules

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Rules:

{Milk,Diaper} → {Beer} (s=0.4, c=0.67)
{Milk,Beer} → {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} → {Milk} (s=0.4, c=0.67)
{Beer} → {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} → {Milk,Beer} (s=0.4, c=0.5)
{Milk} → {Diaper,Beer} (s=0.4, c=0.5)

Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
  - ➢ 이 때문에, 지지도(support)와 신뢰도(confidence)를 분리하여 마이닝 할 필요 있음

Contents

Frequent Itemset Generation &
Apriori Algorithm

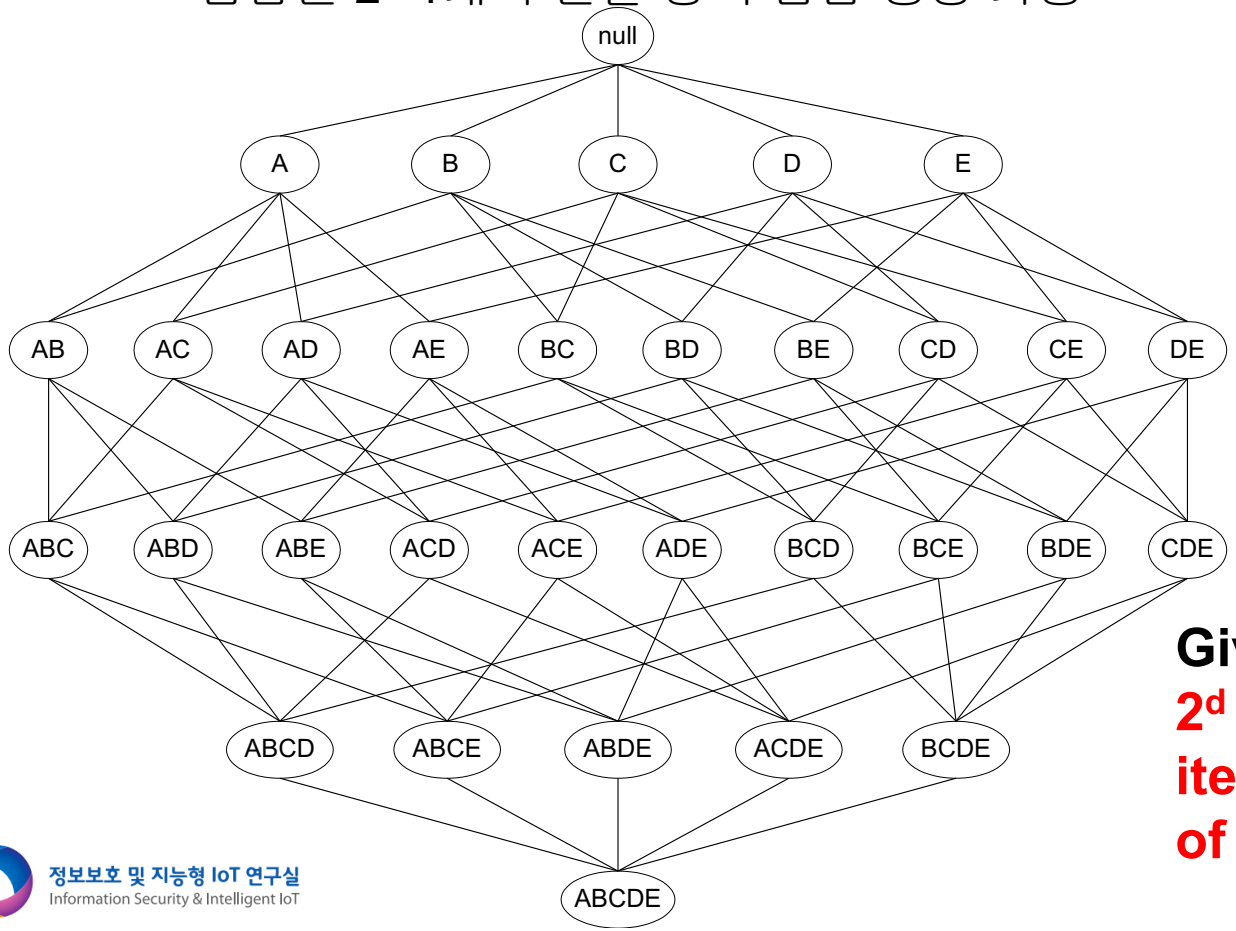
Mining Association Rules

- Transaction DB에서 연관 규칙을 찾아야 함
- 연관규칙을 만들기 위한 2단계 접근법(Two-step approach):
  1. 빈발 항목집합 생성(Frequent Itemset Generation)
     - Transaction DB에서 많이 존재하는 빈발 항목집합을 찾아야 함
     - 빈발 항목집합: support ≥ *minsup*을 만족하는 항목집합
  2. 연관 규칙 생성(Rule Generation)
     - 각 빈발 항목집합을 두 개의 항목집합으로 분리하여 confidence ≥ *minconf*를 만족하는 연관규칙들을 도출함
     - (Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset)

- 여기서, Transaction DB에서 빈발 항목집합을 찾아내는 과정은 computationally expensive 함

Frequent Itemset Generation

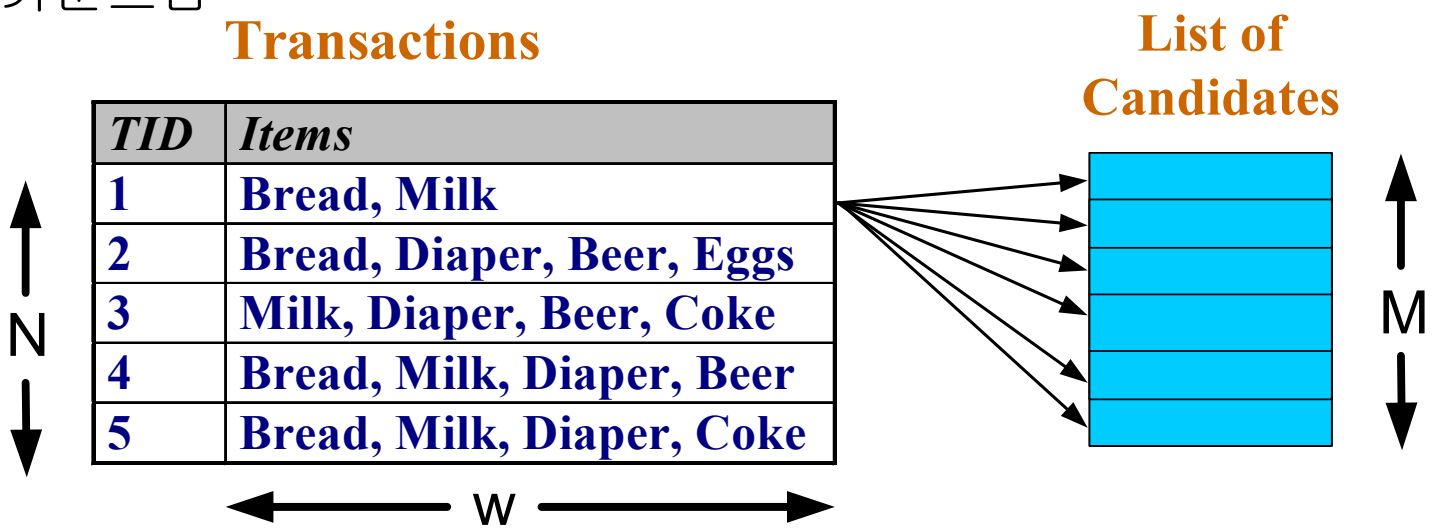
- **Lattice 기반 항목집합 열거**
  - 아래의 lattice(격자) 구조는 모든 가능한 항목집합 목록 열거에 도움됨
  - I={a,b,c,d,e}에 대한 항목 집합 격자. D개의 항목을 포함하는 데이터 집합은 2^d-1개의 빈발 항목 집합 생성 가능



Given d items, there are 2^d possible candidate itemsets (The number of subsets)

Frequent Itemset Generation

- Brute-force approach:
  - Lattice의 모든 itemset은 candidate frequent itemset이 됨
  - Transaction 데이터베이스를 스캔하면서, 각 후보에 대해 support 계산 및 카운트함

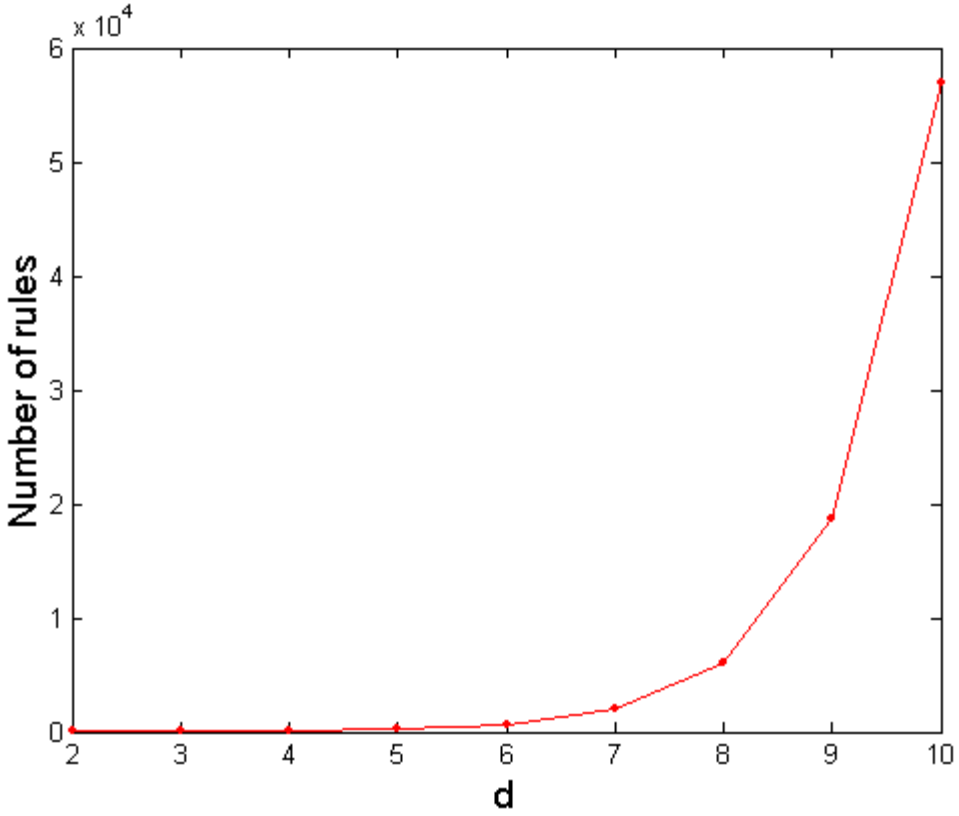


  - 모든 후보에 대해 각 transaction을 매치함
  - 복잡도~ O(NMw) => Expensive since M = 2^d !!!

(M:candidate frequent itemset의 개수, N:Transaction 수, w:최대 transaction 폭)

Computational Complexity

- 항목이 d개 주어졌을 때,
  - 가능한 항목집합의 개수 = 2^d
  - 가능한 연관규칙의 개수 = $3^d - 2^{d+1} + 1$



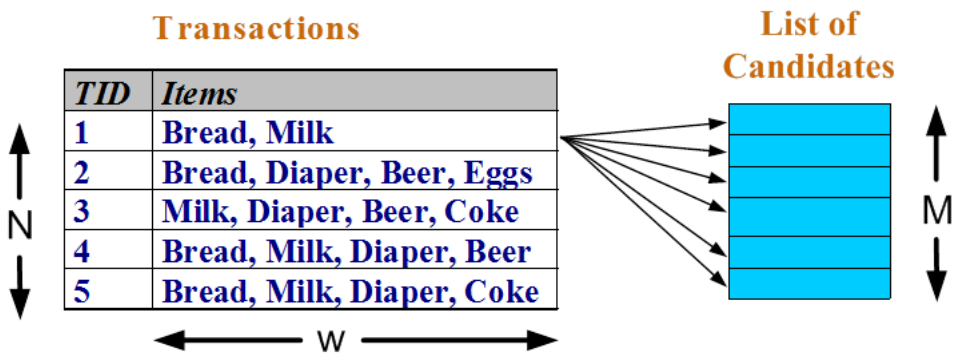
$$R = \sum_{k=1}^{d-1}\left[\binom{d}{k} \times \sum_{j=1}^{d-k}\binom{d-k}{j}\right]$$

$$= 3^d - 2^{d+1} + 1$$

If d=6, R = 602 rules

Frequent Itemset 생성 전략

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - **Use pruning techniques to reduce M**
- Reduce the number of transactions (N)
  - Reduce size of N as the size of itemset increases
  - Using the **DHP(Direct Hashing & Pruning)** and **vertical-based mining algorithms**
- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction



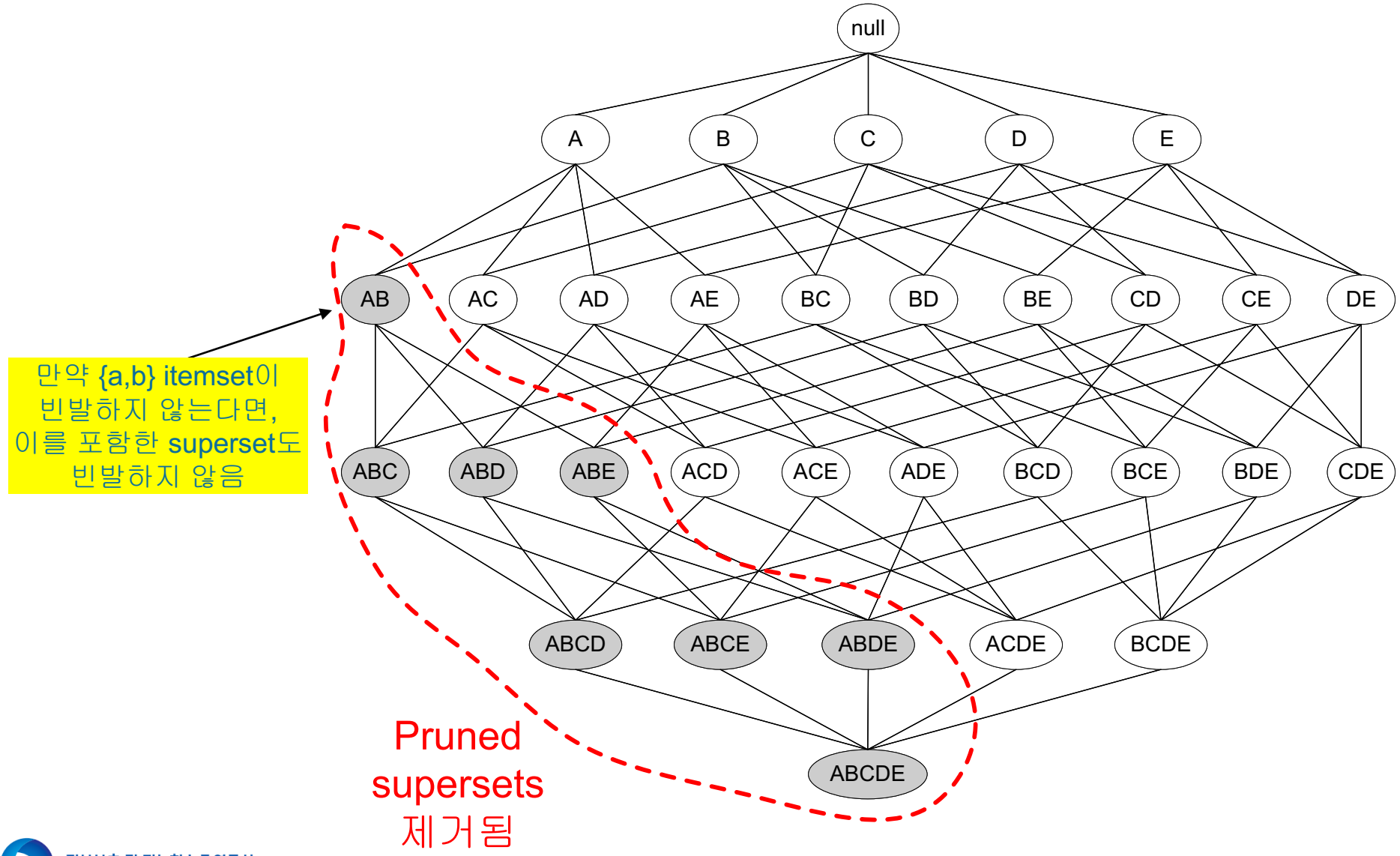
Reducing Number of Candidates

- **Apriori principle:**
  - 어떤 항목집합이 빈발하다면, 그 항목집합의 모든 부분집합도 빈발함
  - 예: {Milk, Bread, Diaper}가 빈발 항목집합이면, 이의 부분집합인 {Milk, Bread}, {Bread, Diaper} 등도 빈발 항목집합이다.
- Apriori principle holds **due to the following property of the support measure:**

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  - 어떤 항목집합의 지지도는 그 부분집합들의 지지도를 넘을 수 없다!
    - ◆즉, 어떤 짧은 서브 패턴이 자주 나오지 않는다는 것을 알고 있다면, 이 서브패턴에서 아이템이 더 붙어서 나오는 수퍼 패턴도 더 자주 나오지 않음
  - 이는 지지도가 anti-monotone 성질을 가지기 때문이다. (a > b ➔ f(a) < f(b))

Illustrating Apriori Principle



Illustrating Apriori Principle

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)

(1) Coke과 Eggs는 상대적으로 빈도가 적으므로 drop됨 (3이하이면 drop)

Minimum Support = 3

(2) Frequent itemset의 항목이 (1)에서 4개이므로, 여기서 2 항목 itemset을 만듦. 4C2 = 6

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(3) 다시 3이하인, 두개 itemset을 drop한 후, 3 itemset을 만듦

Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	3

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3 = 41$$ (Brute-force)

With support-based pruning,

6 + 6 + 1 = 13로 줄어듦 (A Priori)

Illustrating Apriori Principle

Sup_{min} = 2

Database

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

Candidate itemset

C_1

1st scan

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

Frequent itemset

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3rd scan

L_3

Itemset	sup
{B, C, E}	2

Frequent 3-itemset 생성

- {A, C, E}는 없음 Why? {A,E}가 L2에 없으므로
- {B, C, E}는 있음. {B,C} {C,E} {B,E} 모두 L2에 있으므로

The Apriori Algorithm (Pseudo-Code)

C_k: Candidate itemset of size k

L_k : frequent itemset of size k

L_1 = {frequent items};

for (k = 1; L_k !=∅; k++) **do begin**

  C_{k+1} = candidates generated from L_k;

  for each transaction t in database do

    increment the count of all candidates in C_{k+1} that are contained in t

  L_{k+1} = candidates in C_{k+1} with min_support

  end

return $\cup_k L_k$;

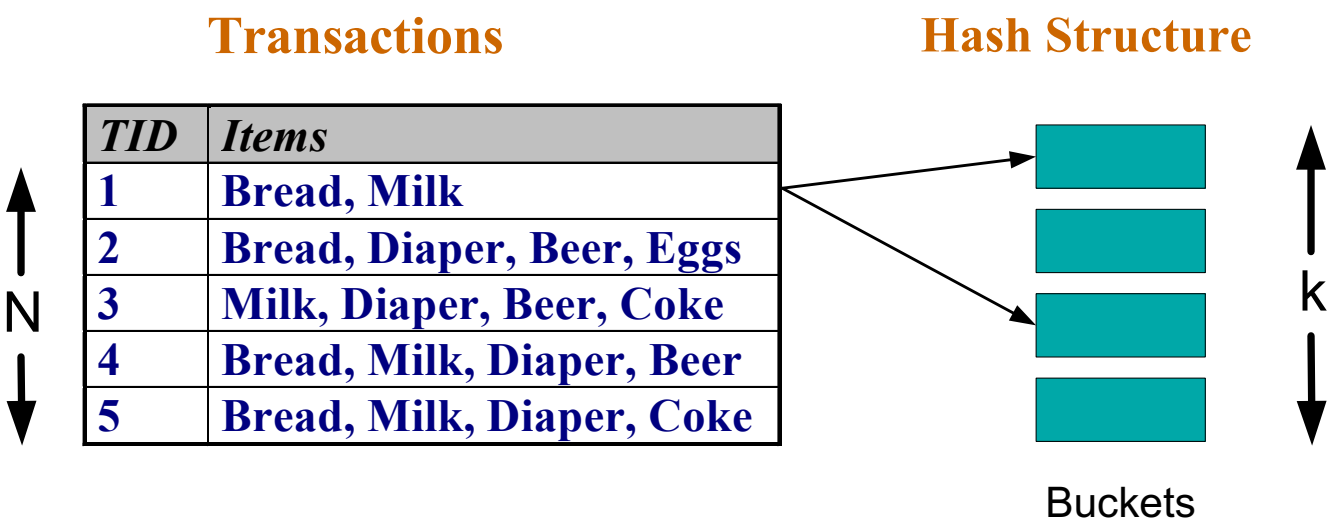
Implementation of Apriori

- How to generate candidates?
  - Step 1: self-joining L_k
  - Step 2: pruning
- Example of Candidate-generation
  - L_3={*abc, abd, acd, ace, bcd*}
  - Step 1: self-joining: L_3*L_3
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Step 2: Pruning:
    - *acde* is removed because *ade* is not in L_3
  - C_4 = {*abcd*}

C_{k+1} = candidates generated from L_k;

Reducing Number of Comparisons

- Candidate counting:
  - 빈발항목이라고 결정내리기 위해선 먼저 candidate itemset에서 support 값을 계산해야 함
  - Support 값 계산을 위해선 Transaction DB와 모두 비교 필요 → 높은 계산 복잡도 → Candidate을 hash structure에 저장하여 효율적 비교
    - Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets
    - That is, we use a hash tree structure to reduce the number of candidates in C that are checked for a data-sequence



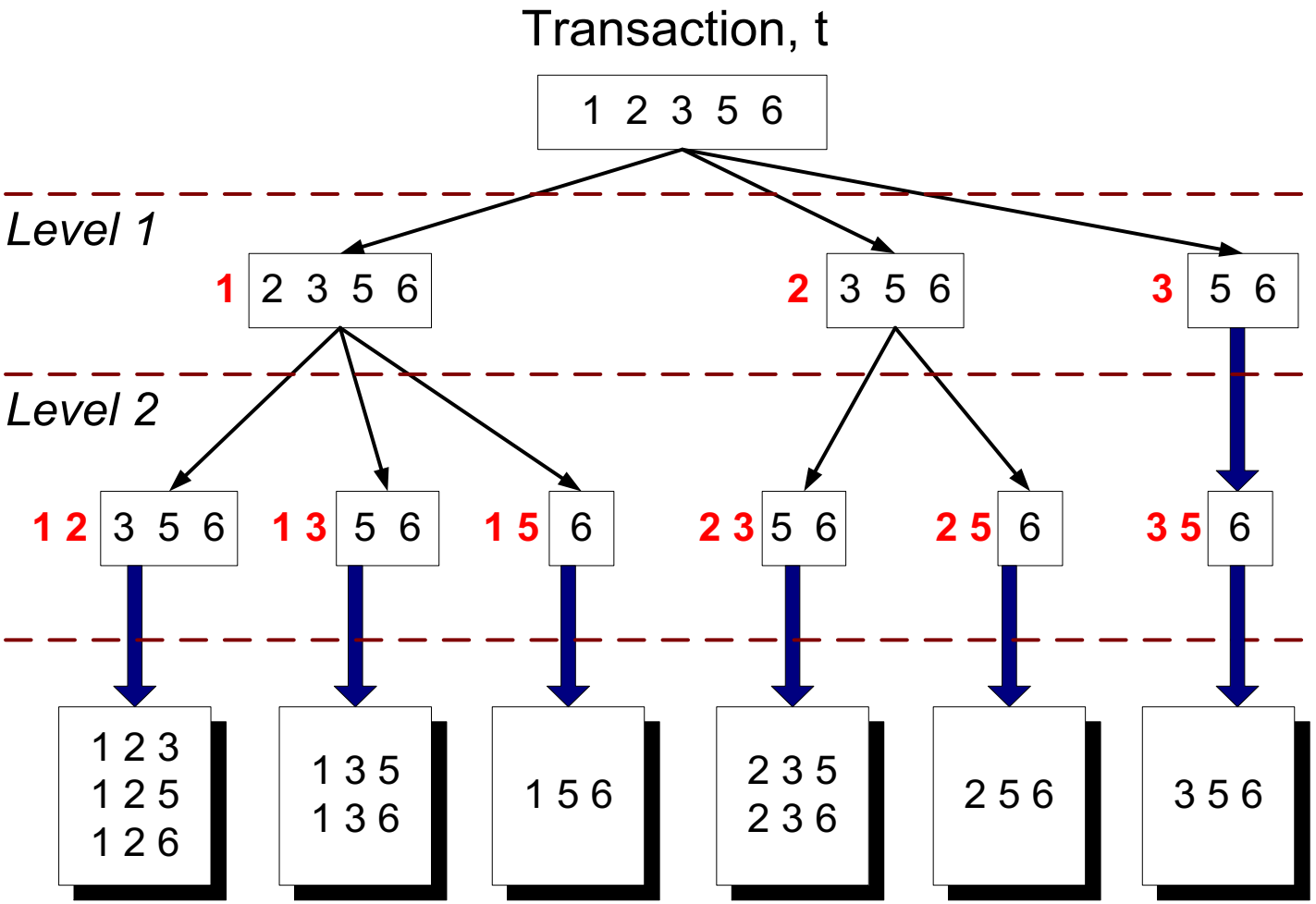
How to Count Supports of Candidates?

- Why **counting supports of candidates** a problem?
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
- Method:
  - Candidate itemsets are stored in a *hash-tree*
  - *Leaf* node of hash-tree contains a list of itemsets and counts
  - *Interior* node contains a hash table
  - *Subset function(hash function)*: finds all the candidates contained in a transaction

Support Counting(지지도 계산) 방식

Transaction t ={1,2,3,5,6}에 속한 3-itemset을 모두 열거하는 체계적 방법

Transaction으로부터 3-itemset을 열거한 후, 후보항목(Candidate list) 각각과 비교하여, 존재하면 해당 후보 항목 지지도 카운트는 증가됨



Support Counting using a Hash Tree

- Candidate itemset이 주어지면→ Hash tree에서 서로 다른 bucket에 나눠 저장됨
  - 이때 지지도 계산시, 각 transaction에 포함된 itemset도 그것들에 적합한 bucket들로 해쉬됨
  - 즉, transaction으로부터 itemset을 hash function 규칙에 따라 hash tree의 각 leaf 노드로 할당함
  - 이로써, transaction에 속한 각 itemset을 각 itemset마다 비교하는 대신, 아래 그림처럼 오직 같은 bucket에 candidate itemset과 비교함

(1) Candidate itemset은 hash tree에 나눠 저장됨

(2) Transaction ID:2,3,4는 Hash Tree의 왼쪽 leaf node (bucket)에만 비교됨

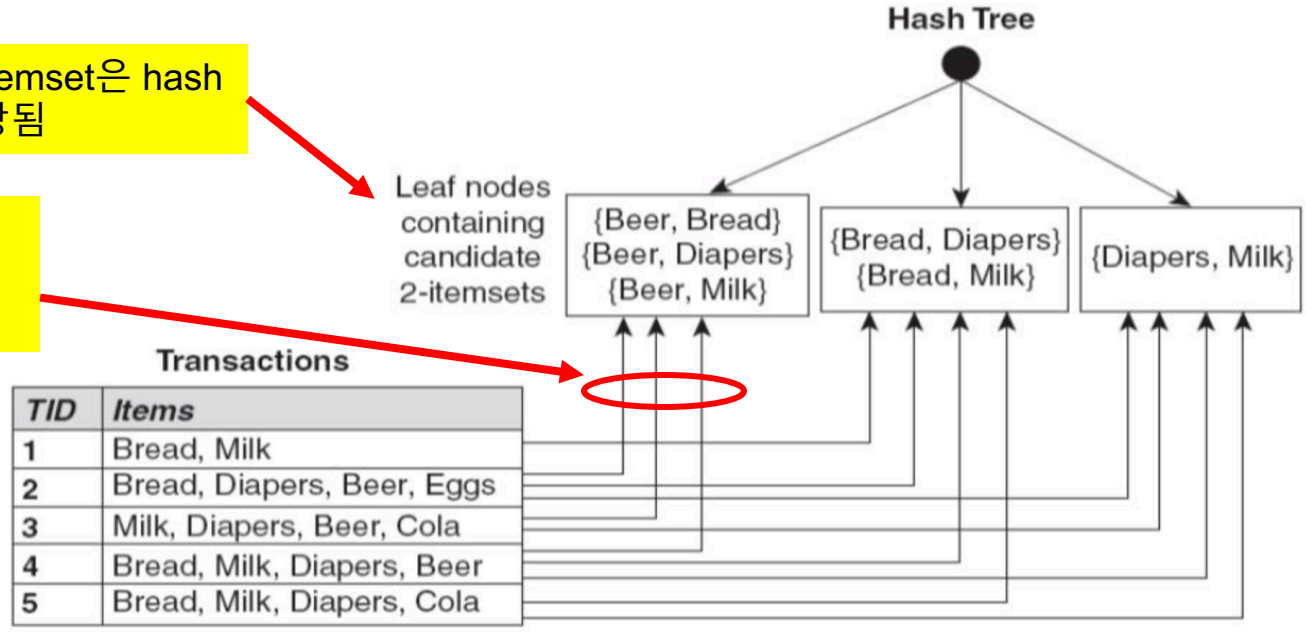


Figure 6.10. Counting the support of itemsets using hash structure.

Support Counting using a Hash Tree

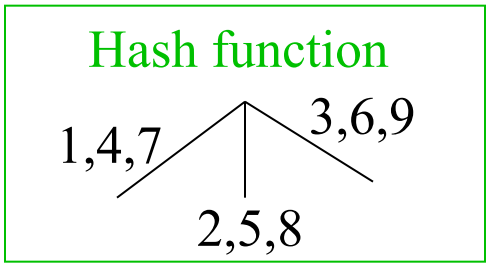
Suppose you have 15 candidate itemsets of length 3:

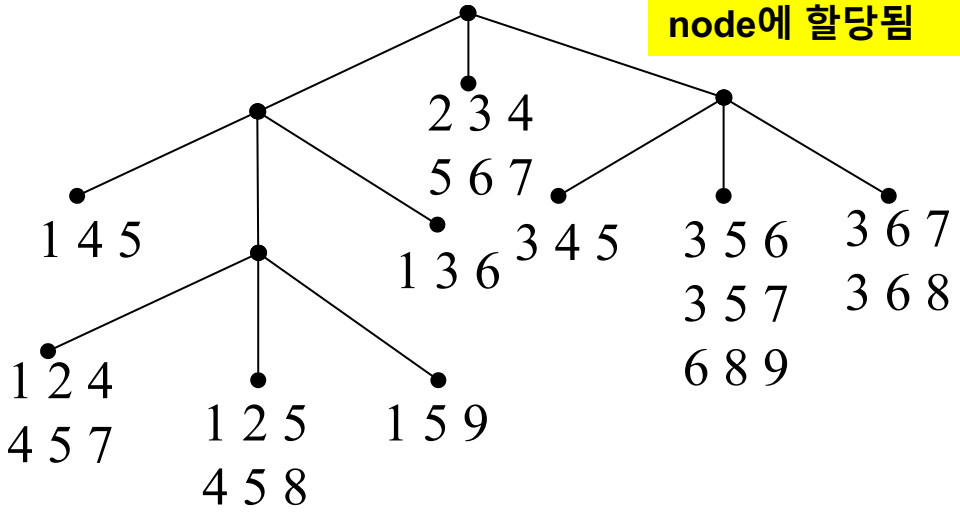
{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

You need:

- **Hash function**
- **Max leaf size: max number of itemsets stored in a leaf node (if number of candidate itemsets exceeds max leaf size, split the node)**

15 개의 candidate itemsets이 leaf node에 할당됨

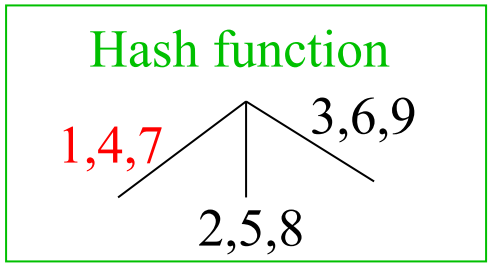


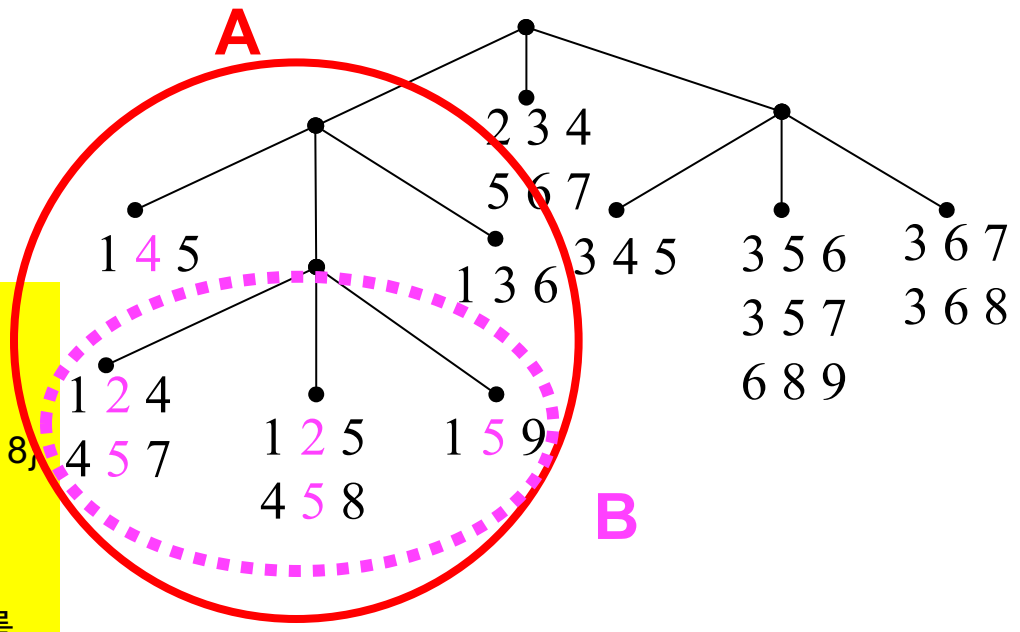


Support Counting using a Hash Tree

Suppose you have 15 candidate itemsets of length 3:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

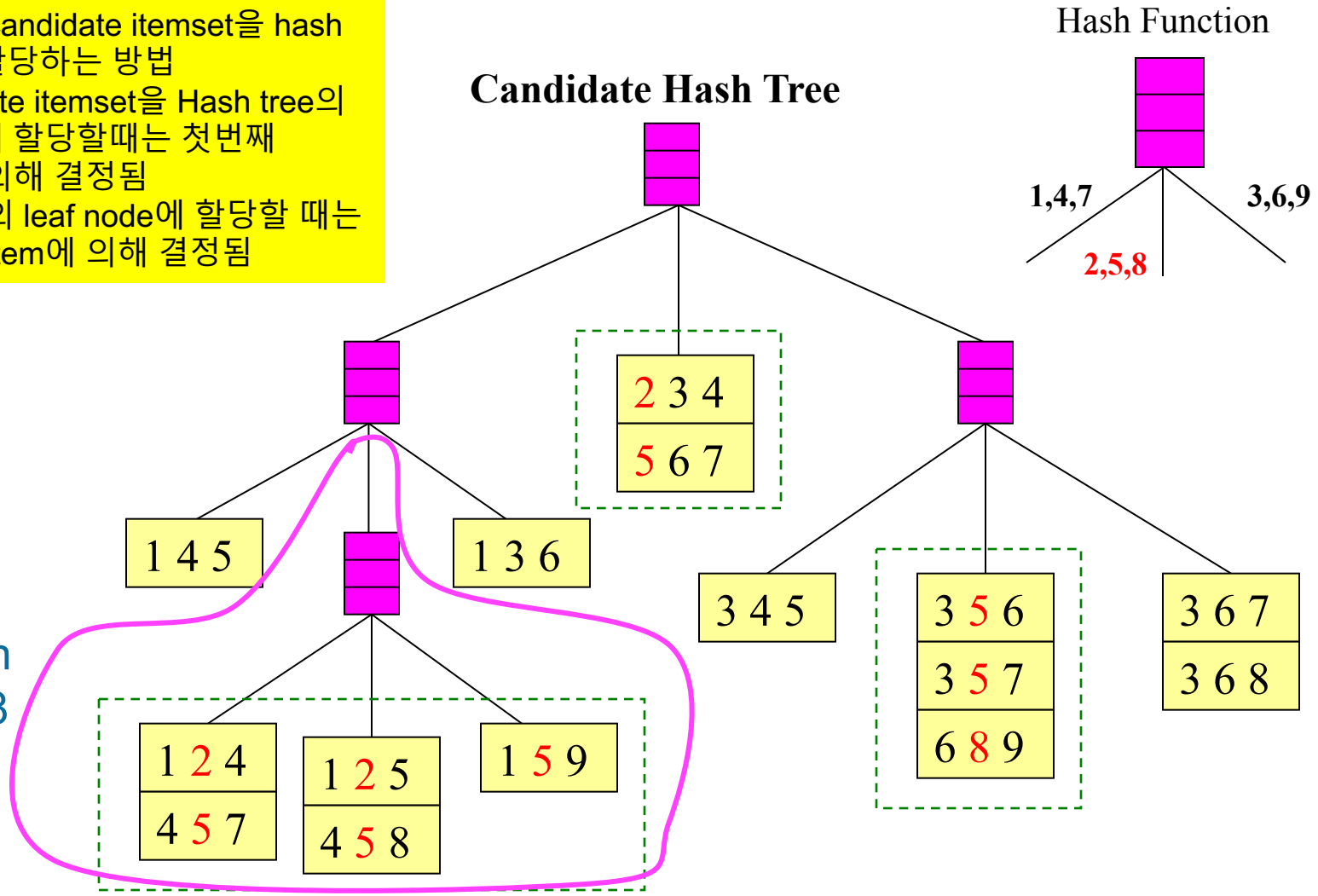




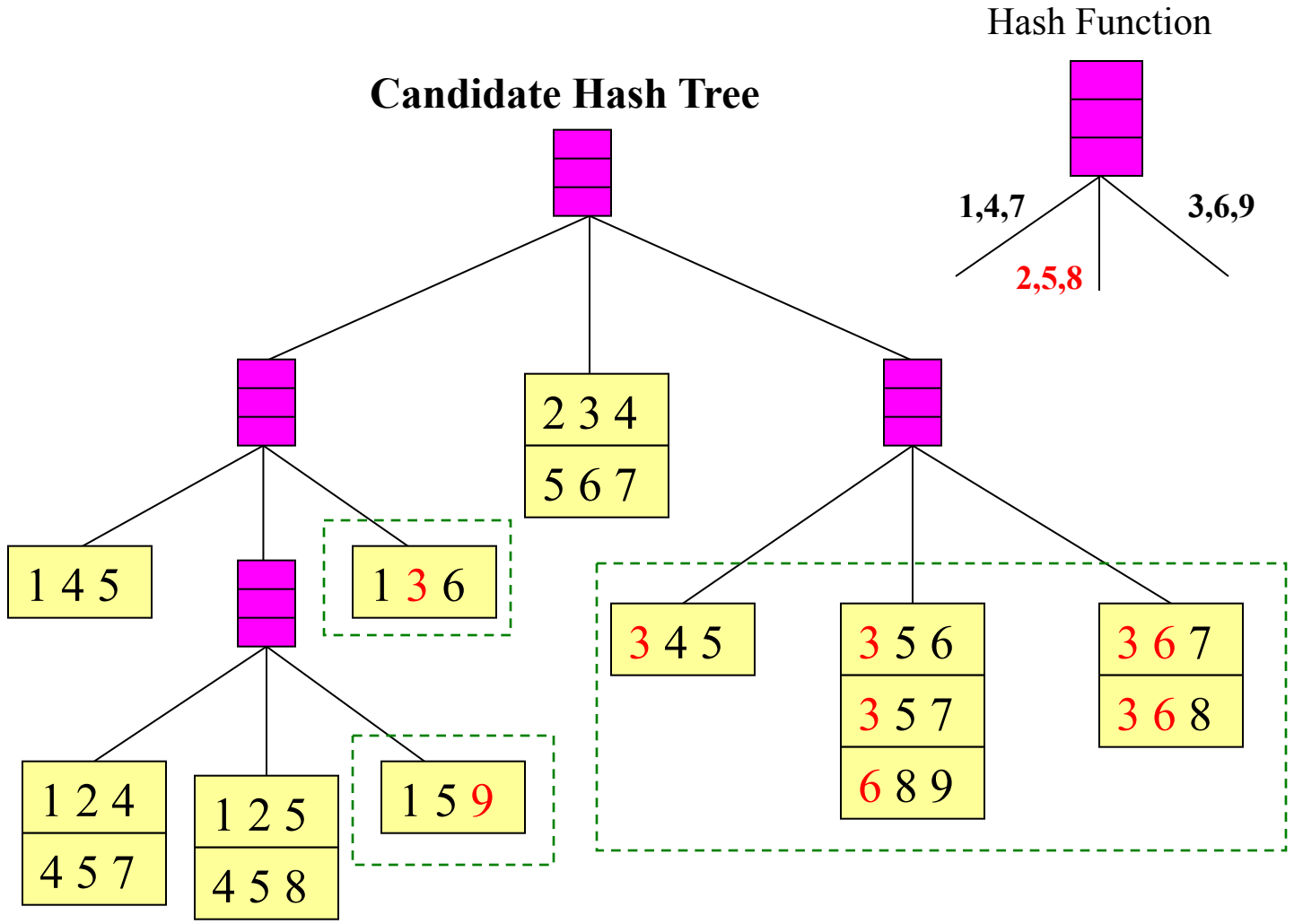
- candidate itemsets에서 1,4,7로 시작되는 것은 A 가지에 모두 할당되어야 함
- 즉, {1 4 5} {1 2 4} {4 5 7} {1 2 5 } {4 5 8} { 1 5 9} {1 3 6}. 총 7개의 itemset이 A 가지에 할당됨
- 각 leaf 노드에 최대 3개만 할당가능하다면 Hash Tree는 depth를 늘림 → B tree 추가됨
- B tree에서는 두번째 값이 2,5 8것을 갖게 되고, 세번째 값에 따라 왼쪽, 가운데, 오른쪽 node로 분배됨

Hash tree

- 주어진 candidate itemset을 hash tree에 할당하는 방법
- Candidate itemset을 Hash tree의 level 1에 할당할때는 첫번째 item에 의해 결정됨
- Level 2의 leaf node에 할당할 때는 두번째 item에 의해 결정됨

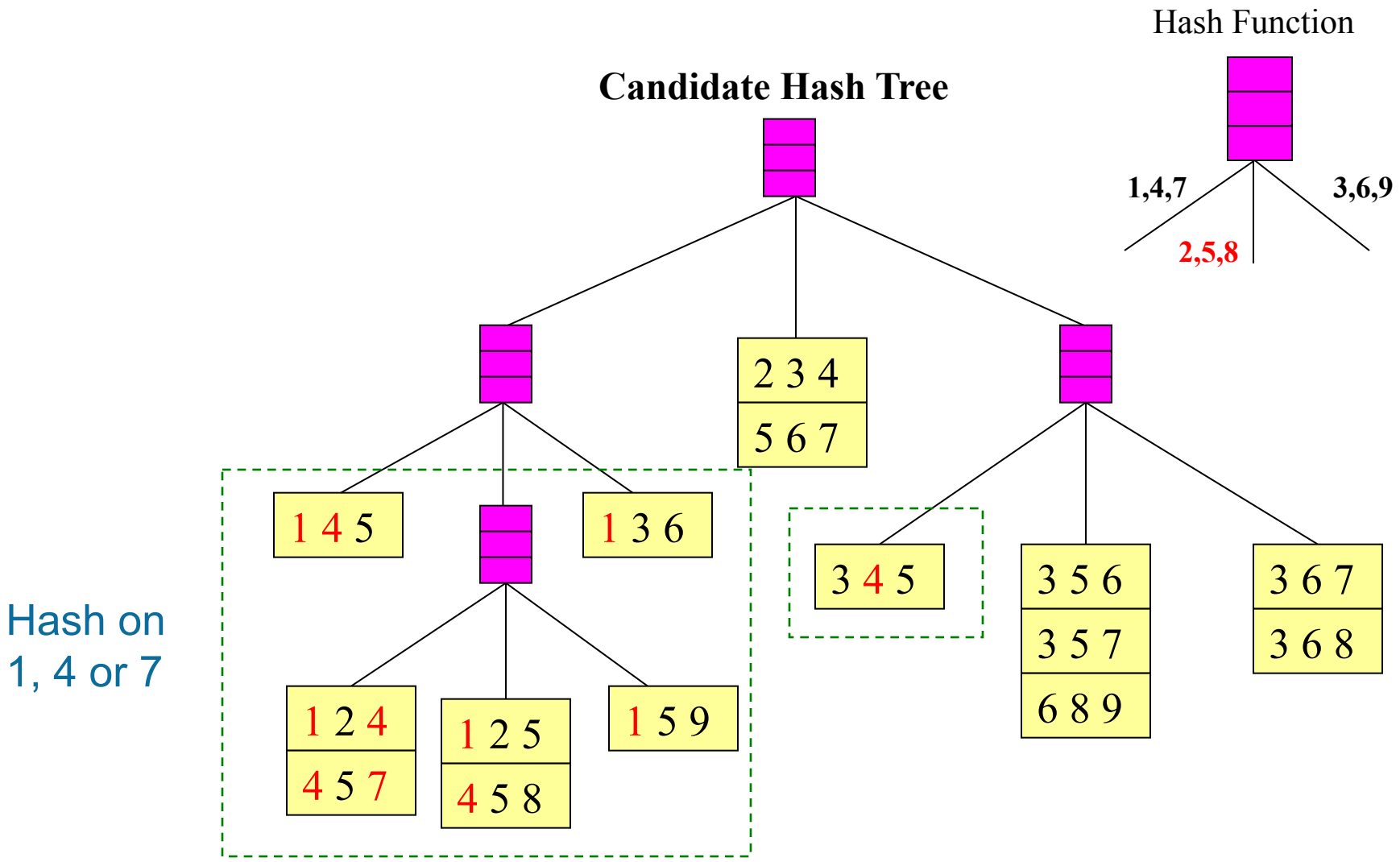


Hash tree



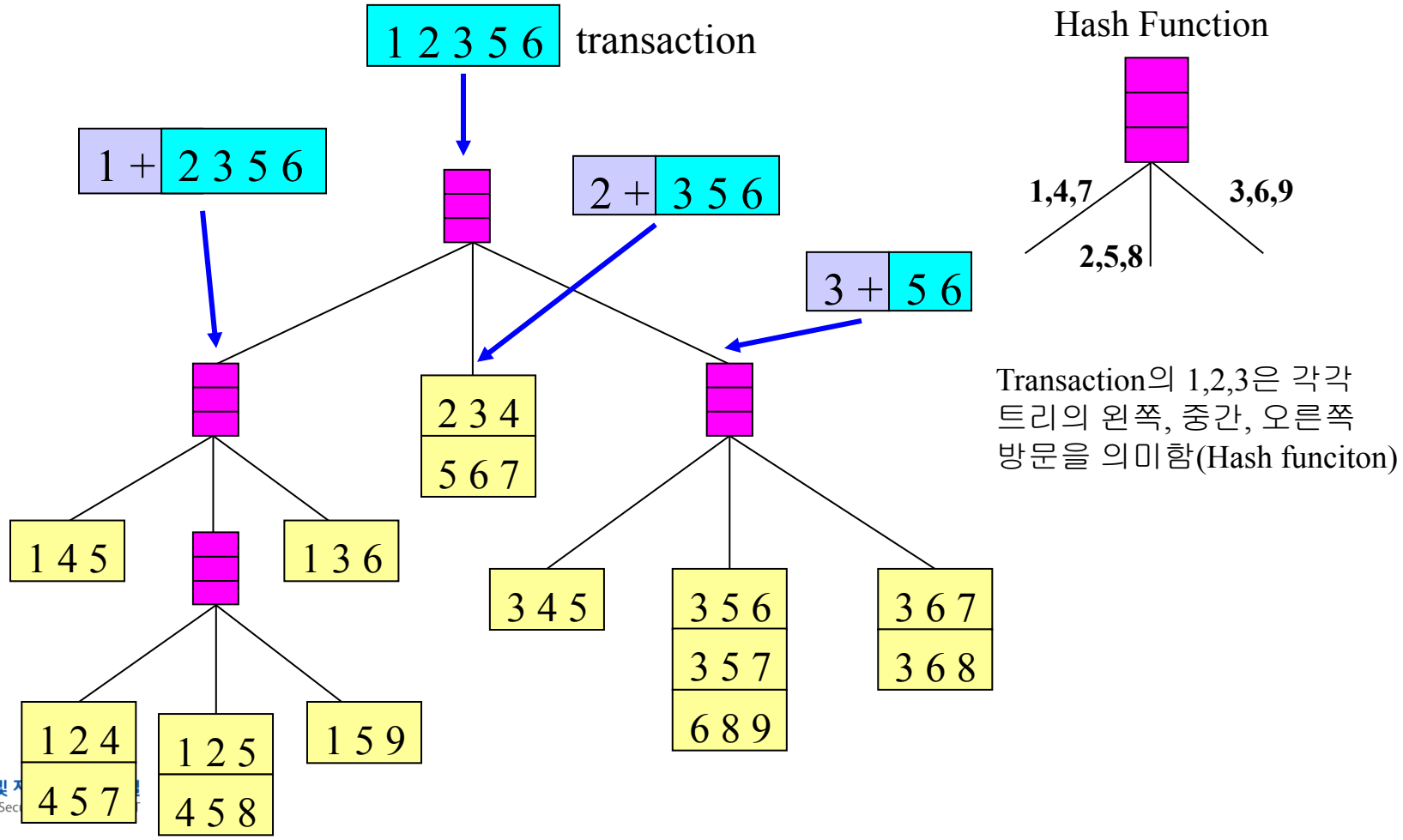
Hash on
3, 6 or 9

Hash tree

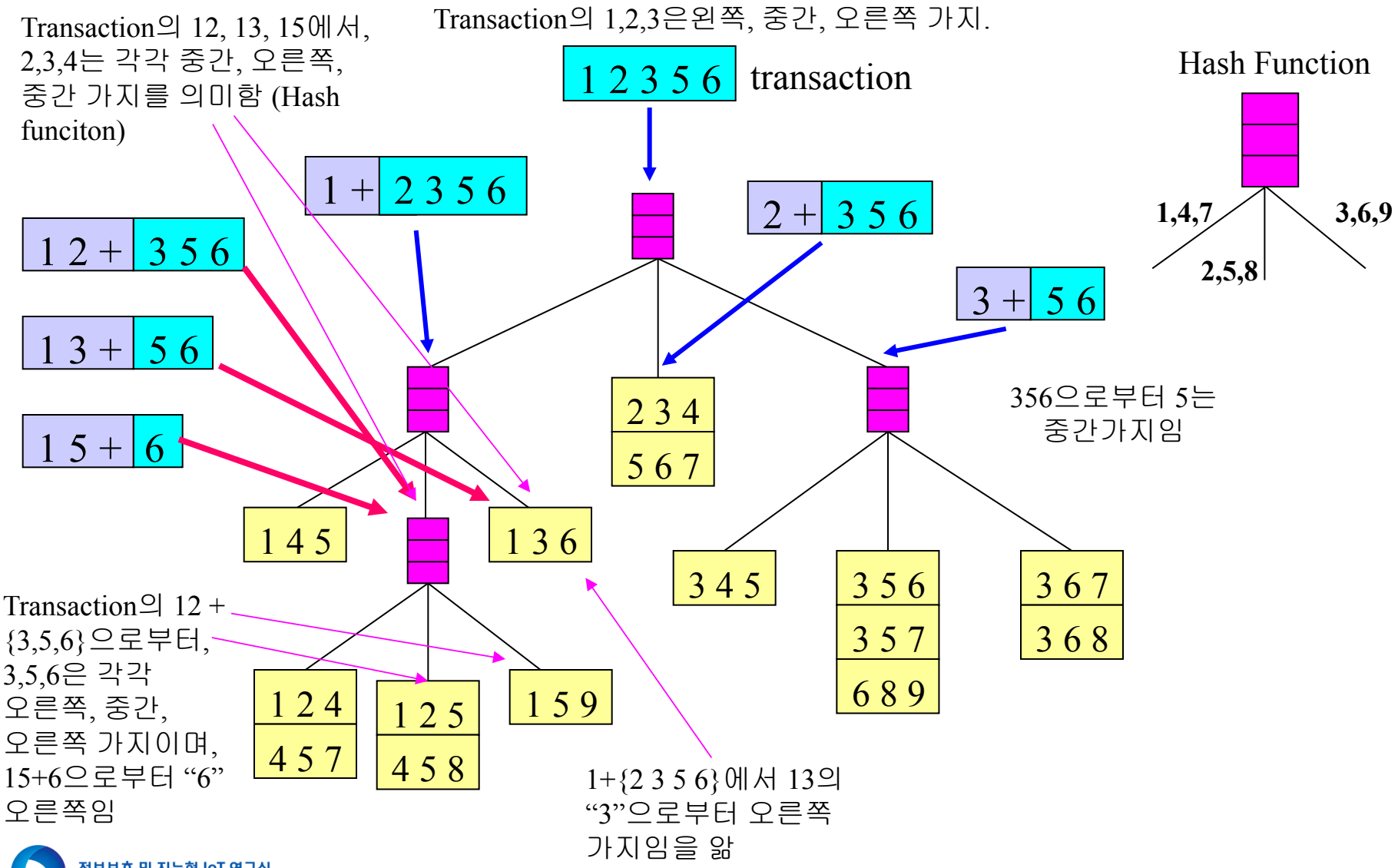

Hash Function
1,4,7
2,5,8
3,6,9
Candidate Hash Tree
1 4 5
1 3 6
2 3 4
5 6 7
3 4 5
3 5 6
3 5 7
6 8 9
3 6 7
3 6 8
1 2 4
4 5 7
1 2 5
4 5 8
1 5 9
Hash on
1, 4 or 7

Support Counting using a Hash Tree

- Transaction {1 2 3 5 6} 은 아래처럼 됨



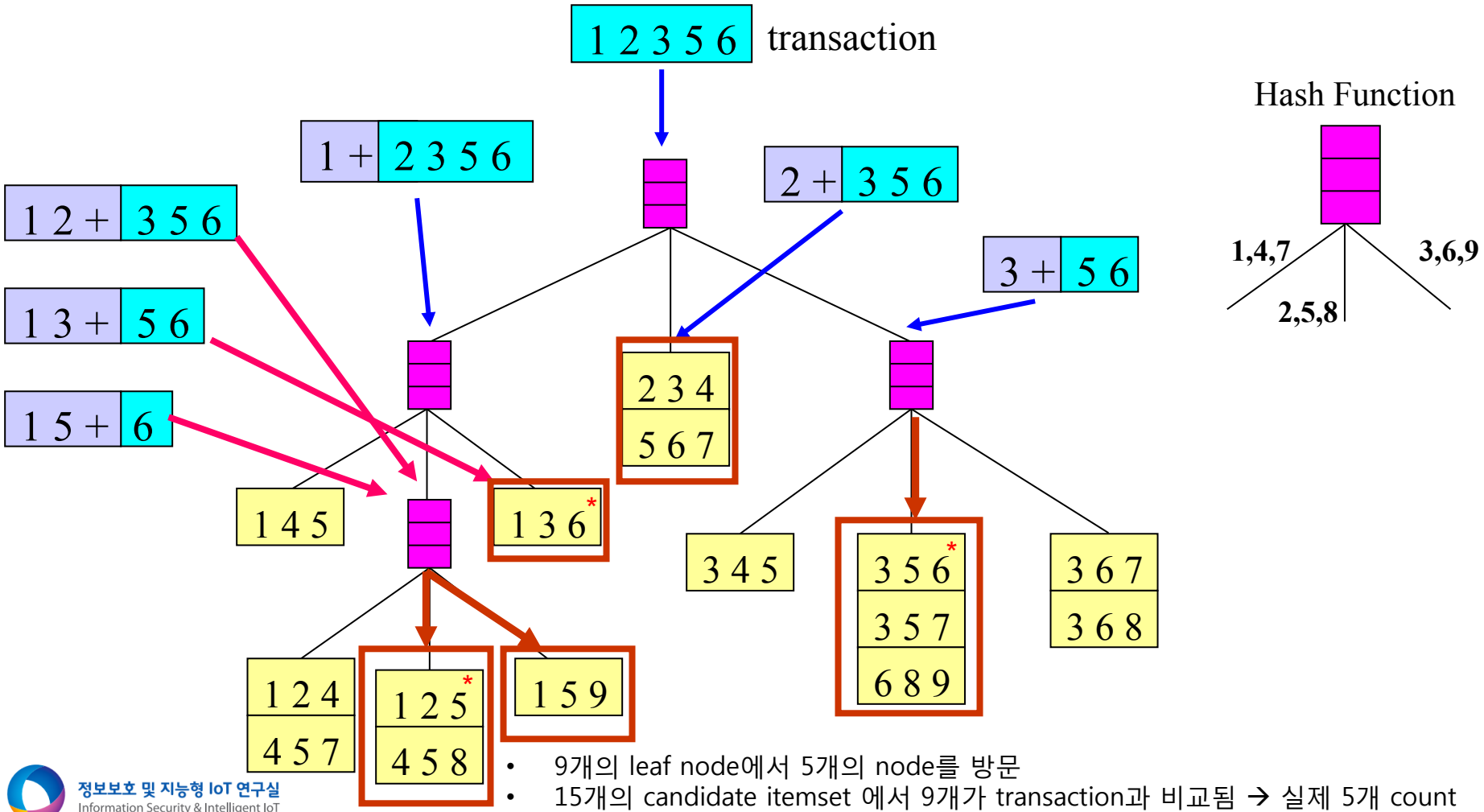
Support Counting using a Hash Tree



Support Counting using a Hash Tree

15 candidate itemsets of length 3:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}



- 9개의 leaf node에서 5개의 node를 방문
- 15개의 candidate itemset 에서 9개가 transaction과 비교됨 → 실제 5개 count 증가됨